# Hw3-1

2023年3月21日 12:03

20. $\int_{-\infty}^{+\infty} f(x)dx = \frac{3}{2}a + b = 1$

$\int_1^2 ax\,dx = \int_2^3 b\,dx \Rightarrow \frac{3}{2}a = b$

$\therefore \begin{cases} a = \frac{1}{3} \\ b = \frac{1}{2} \end{cases}$

21. (1) $\int_{-\infty}^{+\infty} f(x)dx = a \cdot \arctan x \Big|_{-\infty}^{+\infty}$

$= \pi a = 1$

$\therefore a = \frac{1}{\pi}$

(2) $F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{-\infty}^{x} \frac{1}{\pi} \cdot \frac{1}{1+t^2}dt$

$= \frac{1}{\pi} \cdot \arctan t \Big|_{-\infty}^{x}$

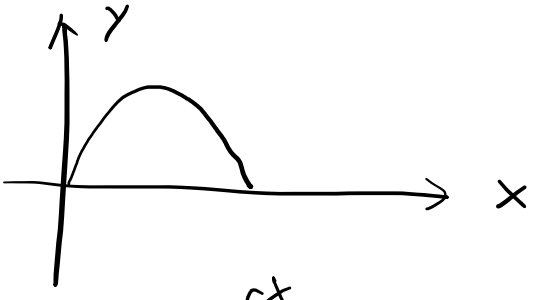
$= \frac{\arctan x + \frac{\pi}{2}}{\pi} = \frac{\arctan x}{\pi} + \frac{1}{2}$

(3) $P(|X|<1) = P(-1<X<1)$

$= F(1) - F(-1)$

$= \frac{1}{2}$

22.




$F(x) = \frac{\int_0^x (2t-t^2)dt}{\int_0^2 (2x-x^2)dx} \quad (0 \le x \le 2)$

$$=\frac{3}{4}x^2-\frac{1}{4}x^3$$

$$\therefore F(x)=\begin{cases}\frac{3}{4}x^2-\frac{1}{4}x^3, & 0\le x\le 2\\ 0, & \text{other}\end{cases}$$

27. 取 $a=0, b=x$

$$F(x)=\int_0^x f(t)dt = F(x)-F(0)$$

$$=k(x-0)$$

$$=kx$$

$$\therefore f(x)=F'(x)=k \text{ 为常数}$$

$$\therefore X\sim U(0,1)$$

28. $$f(x)=\begin{cases}0, & x<0\\ e^{-x}, & x\ge 0\end{cases}$$

$$F(x)=\begin{cases}0, & x<0\\ 1-e^{-x}, & x\ge 0\end{cases}$$

(1) $P(X>2)=1-P(X\le 2)$

$$=1-F(2)$$

$$=1-(1-e^{-2})=e^{-2}\approx 13.53\%$$

(2) $P(X>2+2\mid X>2)=P(X>2)=13.53\%$